

Improved boundary conditions have been derived using analytic solutions of the wave propagation problem in a stratified medium. (4) Ballooning modes. Fluid ballooning modes which are localized to streamlines in classical fluids have been studied. It was found that stable flow fields are the exception rather than the rule. The effects of boundary conditions and rotation on plasma pressure driven ballooning modes have also been studied. (5) Adaptive finite element MHD code. An adaptive mesh MHD code is being studied capable of resolving current sheets, which can form at arbitrary locations in three dimensional reconnecting coronal magnetic fields.

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Work Description

Intermediate Time Scale Magnetic Reconnection

We plan to complete a numerical study of intermediate magnetic reconnection time scales. This will be in collaboration with Dr. D. W. Longcope, now at the University of California, Berkeley. The work, in progress, was originated at N.Y.U. It is a study of magnetic reconnection driven by the doubly periodic coalescence instability (Longcope and Strauss, 1993). If the plasma resistivity is varied in the simulations, there is an enormous variation in the rate of magnetic reconnection. For moderate magnetic Reynolds numbers (Lundquist number) less than about 3000, the reconnection rate is almost independent of resistivity. This was identified previously as the fast reconnection regime (Strauss, 1993). For larger Lundquist numbers greater than about 3000, the reconnection rate slows down, and appears to be approaching a Sweet - Parker regime. Our simulations would provide the first published description of the transition from the fast to slow reconnection. The reason we are able to do this is because of the favorable properties of the doubly periodic coalescence instability, which has a low transition Lundquist number. The results of our study should help explain the temporal behavior of solar flares and the wide range of time scales in flares.

Adaptive Unstructured Mesh MHD Code

We plan to continue the development of an adaptive finite element MHD code (Strauss, 1993). This code uses an unstructured mesh of triangles in two dimensions, and prisms in three dimensions. The mesh can be adaptively refined in order to resolve fine structure. The need for this adaptivity is present in MHD simulations of solar active regions, in which MHD instability and photospheric driving cause the formation of highly localized and intense current layers, the sites of magnetic reconnection. The adaptive unstructured mesh is a promising approach to correctly resolving such intermittent structures. The code presently uses piecewise linear elements. We plan to test higher order discretizations, compatible with the order of the MHD equations. We also plan to parallelize the code on an IBM RISC cluster.

3D Coalescence Instability

Solar flares can occur when the stored magnetic free energy is rapidly released by magnetic reconnection.

Magnetic reconnection is fairly well understood in two dimensional theory and numerical simulations in which there is an ignorable coordinate. Reconnection occurs as conducting fluid flows across a magnetic separatrix, which divides topologically distinct field lines from each other. Resistive dissipation has a negligible effect, except in a highly localized layer, where an intense current density forms.

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In the layer, magnetic energy is released by conversion to kinetic energy, and by Ohmic conversion to heat.

We investigated 3D line tied spontaneous reconnection. We have found a new version of the 2-D MHD coalescence instability (Longcope and Strauss, 1993) as well as its 3-D, line-tied counterpart (Longcope and Strauss, 1994). The initial, equilibrium magnetic field consists of a nearly constant axial component B_z , and transverse components which form a diamond like pattern of islands, having axial current of alternate sign in adjacent islands. Contours of the equilibrium flux function A_z in the $x - y$ plane are shown in fig. 1a. Islands of like sign are attracted together, which is opposed by line tying. The linear growth rate and linear eigenmode were found analytically. The 3D instability condition is $PL < q_c = 0.854$, where the field line pitch $P = 0.467B\ell^2/A_0$, A_0 is the peak value of A_z , B is the average magnetic field strength, and $2\pi\ell$ is the periodicity length in x and y . The value of $\iota = L/P$ is the magnetic field twist (divided by 2π) of the o - line through each island. The analytic growth rate is in excellent agreement with numerical simulations.

Nonlinear 3-D resistive MHD simulations were carried out with CHTH, a full MHD code using finite difference discretization on a staggered mesh. Boundary conditions are periodic in the $x - y$ plane, and line tied at the ends $z = 0, L$. Starting from an unstable equilibrium, the simulations show a linear, exponential growth of kinetic energy, a nonlinear ideal phase in which the kinetic energy saturates, followed by a phase in which reconnection occurs. The magnetic field is nearly symmetric about the midplane $z = L/2$. Hence, the axial flux A_z in the midplane is a good approximation to a flux function. (In two dimensions, field lines are exactly tangent to contours of constant A_z .) Figs. 1 b-d show the time development of the A_z contours in the midplane. Fig. 1b shows the nonlinear ideal phase. The diamond shaped islands have deformed into pentagonal shapes. An intense current layer formed on the short side of the pentagons, where the diamonds have pressed together.

As reconnection proceeds, the pentagons merge into pairs of islands, in fig. 1c. Finally, the pairs of islands join to form a new pattern of square islands, with size $\sqrt{2}$ larger than initially. If the final state were independent of z , this evolution would release half the energy stored in the initial transverse magnetic field B_x, B_y . Because line tying inhibits the motion near the ends, the final state is three dimensional and the energy release is about 25% of the initial transverse magnetic energy.

An idea of the 3D structure of the current can be seen in Fig. 1e. This shows the current in the lower quadrant of the $x - y$ plane, as a function of z . The two hose like structures at the top and bottom are current channels that were present in the initial equilibrium. They are attracted together by the coalescence instability. In between them is an intense current layer. The current density J in the layer is strongest in the midplane $z = L/2$, and gets weaker toward the endplanes. As in earlier 3D driven coalescence simulations, the current layer forms a twisted ribbon.

The other structures in fig. 1e are pieces of other current channels.

The midplane plots of A , are quite suggestive of 2D reconnection, but to demonstrate reconnection in 3D it is necessary to trace field lines. In ideal MHD, magnetic field lines are “frozen” to the fluid. Reconnection may be loosely defined as the separation or slippage of field lines from fluid elements, by a distance comparable to the scale of the system (in order to distinguish it from resistive diffusion). This is clearly shown in fig. 2a-c. Field lines originating at $z = 0$ are shown in projection on the $x - z$ plane. Initially, the field lines lie on straight helical flux ropes. In fig. 2a, the field lines are shown in the ideal phase of the coalescence instability. The field lines follow kinked helices, which still connect the same endpoints at $z = 0, L$ as in the initial state. In the reconnection phase, in fig. 2b, the flux ropes fray in the middle, where the field lines pass near the most intense part of the current layer. Finally, in fig. 2c, the flux ropes have reconnected, and join locations on the end planes widely separated from the initial positions. The fluid elements on the endplanes have not moved at all, because of line tying. The displacement of the field lines shows that reconnection has occurred. This shows that while line tying somewhat inhibits reconnection, it does not prevent it.

Line Tied Gravitational Ballooning Instability

In the theory of the line tied gravitational ballooning instability (Strauss and Longcope, 1994) found a new two dimensional prominence model, which generalizes the Kippenhahn Schlüter model. We then analyzed its stability and found that the instability condition can be expressed in terms of the angle between the magnetic field and the prominence axis. As the angle decreases, the prominence gets more unstable. This is very suggestive of the data indicating a relationship between magnetic shear in arcades and solar flares. Increasing the shear decreases the angle. Our calculation assumed short wavelengths, so our instability might describe the irregular “rain” falling from prominences.

We would like to use the new finite element code for a study of line tied gravitational ballooning modes. The theory (Strauss and Longcope, 1994) considered only short wavelengths. A simulation could compare with the theory and go on to look at long wavelengths. The long wavelength instability might be responsible for the destabilization of cold dense filaments or prominences, which would initiate large flares or prominence eruptions.

Boundary conditions for the solar corona

The corona is dynamically connected to the lower solar atmosphere (chromosphere and photosphere), through the magnetic field lines which traverse all these regions and extend into the solar interior. In discussing any coronal motion or wave phenomenon, it is necessary, in principle, to couple it to its extension into the lower atmosphere. This, of course, complicates the analysis considerably. It has been the practice to simplify matters by modelling the effects of the chromosphere and photosphere only through their influence on the boundary conditions imposed at

the base of the corona. A popular device is to use the field-line boundary conditions where the lower atmosphere is modelled as a perfect conductor which fixes the position of the feet of the field lines. This approach, unfortunately, is not too accurate because it does not account for coronal waves, in particular Alfvén waves, which are partly transmitted to the lower atmosphere and then get lost in the solar interior.

We have solved this problem in a configuration appropriate for Sunspots. We consider the magnetic field and temperature to be constant in the lower atmosphere. The full wave solution of such a plasma (with stratification of the density) is known in the literature and may be written in terms of hypergeometric (Meijer) functions of the dimensionless variable ζ^2 , where $\zeta = \omega H/V_A$. (ω = frequency, H = density scale height, V_A = Alfvén speed.) Moreover, it is known how to connect asymptotically the solutions for $\zeta^2 \ll 1$ and $\zeta^2 \gg 1$. Note that $\zeta^2 \gg 1$ is deep in the lower atmosphere where solutions behave asymptotically like waves. Our time scale of interest is Alfvén wave bounce time along coronal loops, an order of minutes, with $\omega \lesssim 10^{-1} \text{ sec}^{-1}$. Taking $H = 200 \text{ km}$ and $V_A = 50 \text{ km/sec}$ just below the corona, we get $\zeta^2 < 0.16$. Imposing the so-called "radiation condition", such that each downwards going wave at $\zeta^2 \gg 1$ gets lost, we get relations for the solution at $\zeta^2 \ll 1$, which is the desired boundary condition.

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Ionospheric influence on magnetospheric boundary conditions

The Low-Latitude Boundary Layer (LLBL) is located just inside the magnetopause and represents the narrow region where solar wind particles enter the day-side magnetosphere. The formation of the LLBL is not well understood. A number of mechanisms have been proposed, such as diffusion or magnetic reconnection followed by the interchange instability. The various theories try to account for the density profile across the layer, as well as for the observed velocity profile within the boundary layer which proceeds from local noon towards the magnetotail but also includes a return flow region. A serious difficulty in these theories is that they do not model well the drag exerted by the conducting ionosphere on the Earth's magnetic field convected by the flow. In particular, the magnetic field in the LLBL is mostly assumed in the theories to be dipolar which, in fact, is far from the truth. The question of ionospheric drag is a boundary condition issue and we now describe how to state the boundary conditions such that the influence of the ionosphere will be accounted for.

The width of the LLBL when mapped by the magnetic field to the ionosphere, is comparable to the height of the ionosphere and atmosphere ($\sim 10^2 \text{ km}$) but much smaller than the meridional length scale in the ionosphere ($\sim 10^3 \text{ km}$). Defining the ratio of the two scales as ϵ , it is possible to solve for the electromagnetic fields in the

ionosphere (modelled as a conductor) and the atmosphere (an insulator) as a series expansion in ϵ . The boundary condition for this coupled system, at the Earth, is tied field lines, but this can be "elevated" to the top of the ionosphere and applied to the magnetospheric LLBL fields. Our previous work [*Hameiri and Kivelson, JGR, 1991*] dealt with a rather similar question but was much easier because of the lack of rapid variation across the LLBL width. Nevertheless, the present situation is also doable and we have obtained the desired result, although expressed in a complicated way. We are presently working on special limits where our boundary conditions can assume a simple form.

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Magnetic field effects on ballooning modes in rotating stars

The analogous phenomena to ballooning modes in plasmas are modes localized to streamlines in classical fluids. We have given these modes a thorough analysis and it is apparent that stable flow fields are the exception rather than the rule. Here we are interested in investigating the effect of the presence of a magnetic field on such instabilities. We do not assume, as is common in tokamak stability studies, that the magnetic energy is much larger than the flow energy. For simplicity we consider an axisymmetric configuration with purely toroidal rotation and with either a purely toroidal magnetic field or a tokamak-like field where, however, we only discuss the magnetic center. We also include the effect of gravity. Such a situation may be relevant to the interior of stars and its simplicity allows for the derivation of exact conditions for stability. In the absence of a magnetic field, instability is due essentially to the Rayleigh-Taylor mechanism, and we recover the previously known Høiland stability criterion. If a magnetic field is present the criterion is much more complicated. For a large enough field, all non-axisymmetric modes will be stable. However, there may still be a window of instability for the axisymmetric modes if the Rayleigh-Taylor drive is sufficiently strong. In fact, we derive a necessary and sufficient condition for the stability of axisymmetric modes. (In this simple configuration, axisymmetric ballooning modes are allowed.) While usually, when flow is present, the energy criterion is only sufficient for stability, we show here that our criterion coincides with the energy principle when properly defined. This example suggests how to define an improved energy integral for more general configurations, such that the energy principle will be closer to the stability limit.

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Ballooning modes for finite-length field lines

When a magnetic field intersects the plasma boundary, as in the Earth's magnetosphere or in the solar corona, the electrical properties of the boundary affect the

plasma behavior through the inducement of boundary conditions different from the usual tied-line conditions. This also causes a change in the stability properties and in the spectrum of eigenmodes of the plasma. Here we consider in particular the existence of ballooning modes and the Alfvén resonance phenomenon when magnetic field lines enter the boundary.

By using "singular sequences" as approximate eigenfunctions, we give a mathematical proof for the existence of the ballooning spectrum in such a configuration. This method was used previously for tokamak plasmas, but here it is necessary to add a rapidly varying piece near the magnetic foot points. This addition is necessitated by the presence of evanescent fast magnetosonic waves in that region. As for the persistence of the Alfvén resonance phenomenon, by specifying varying electrical conductivity at the magnetic foot points we can generate different Alfvén frequencies for different field lines on the same pressure surface. We then demonstrate a resonance situation where only one field line is in resonance, unlike the common view which sees the phenomenon as a resonance of an entire magnetic surface. This is in marked contrast with the tokamak situation where the whole pressure surface is in resonance at the same time simply because the same field line covers the surface ergodically.

E. Hameiri (CIMS)

Meetings and Laboratory Visits

We attended the American Physical Society Meeting in November, 1993. We gave presentations at the AGU Meeting, Baltimore, May, 1993. We gave a talk at the Computational Mathematics AFOSR Contractors Meeting, St. Louis, April, 1993. E. Hameiri spent a month at HAO - NCAR in July, 1993.

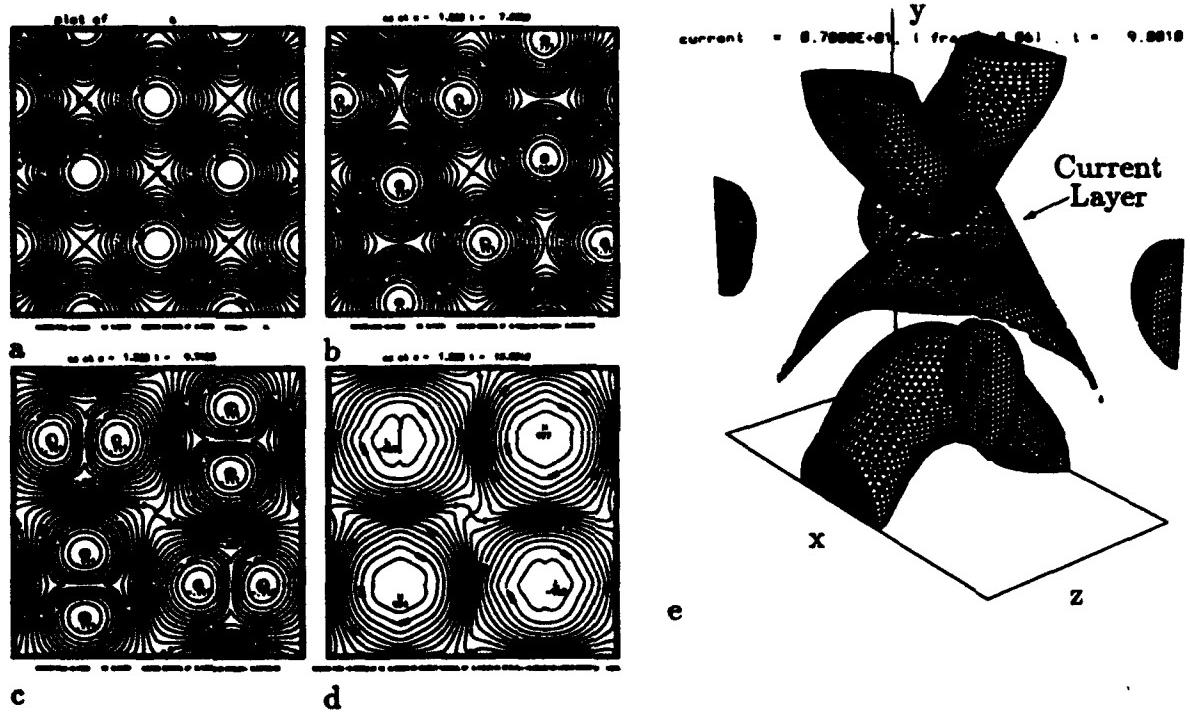


Figure 1: 3D line tied coalescence: (a-d) Flux $A_s(x, y, L/2)$ at times (a) $t = 0$, (b)7, (c) 9.7, (d) 13.5. (e) A 3D isoplot of the current at $t = 9$. The current layer is in the center.

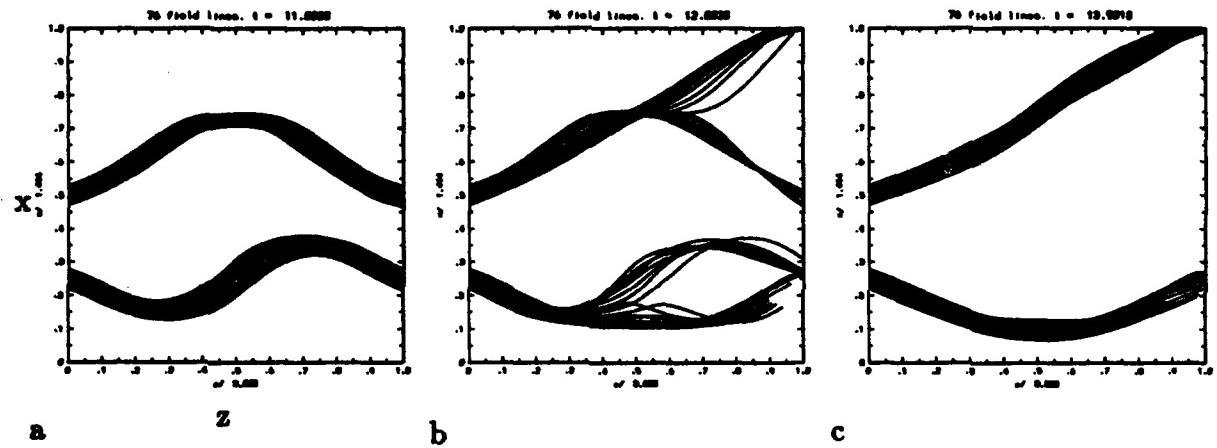


Figure 2: Field line traces from a nonlinear run at times (a) $t = 11.0$; (b) $t = 12.0$; (c) $t = 13.5$. Field lines originate at the same points in the $z = 0$ plane in all graphs. View is a projection onto $x-z$ plane. At time (a), the flux tubes are strongly kinked from their initial straight configuration, but are not reconnected. At time (b), the flux tubes begin to unravel in the middle. At time (c), reconnection is complete.

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